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Kaluza-Klein Cosmological Models with anisotropic Dark energy and Hybrid Expansion Law

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Abstract: In this paper we have studied Kaluza-Klein cosmological model filled with anisotropic fluid in general relativity. The solutions of the Einstein's field equations are obtained under the assumption of Hybrid Expansion Law (HEL). The isotropy of the fluid, space and expansion are examined.

Keywords: Kaluza- Klein space-time, Anisotropic Fluid, Dark Energy, Isotropization, Hybrid Expansion Law.

1. Introduction:

Recent most remarkable observational discoveries have shown that our universe is currently accelerating. This was first observed from high red shift supernova Ia [1-7], and confirmed later by cross checks from the cosmic microwave background radiation[8-9] and large scale structure [10-15]. In Einstein's general relativity, in order to have such acceleration, one needs to introduce a component to the matter distribution of the universe with a large negative pressure. This component is usually referred as dark energy [DE]. Astronomical observations indicate that our universe is flat and currently consists of approximately 2/3 dark energy and 1/3 dark matter. The nature of dark energy as well as dark matter is unknown, and many radically different models have been proposed, such as, a tiny positive cosmological constant, quintessence [16-18], DGP branes [19-20] , the non-linear F(R) models [21-23] , and dark energy in brane worlds, among many others [24-39] see also the review articles[7] and [40], and references therein. As mentioned before, the existence of dark energy fluids comes from the observations of the accelerated expansion of the universe and the isotropic pressure cosmological models give the best fitting of the observations. Although some authors [41] have suggested cosmological model with anisotropic and viscous dark energy in order to explain an anomalous cosmological observation in the cosmic microwave background (CMB) at the largest angles.

By choosing the appropriate value of the scale factor which yields a time-dependent deceleration parameter, many authors have considered the following anastz for the scale factor of the Universe:

$$a = a_0 t^{\alpha} e^{\beta t},$$

where $a_0 > 0$, $\alpha > 0$ and $\beta > 0$ are constants.

This generalized form of scale factor referred as the Hybrid Expansion Law (HEL). It is observed that the HEL leads to the power-law cosmology for $\beta = 0$ and to the exponential-law cosmology for $\alpha = 0$. Thus, the power-law and exponential-law cosmologies are the special cases of the HEL. Therefore, the case $\alpha > 0$ and $\beta > 0$ leads to a new cosmology arising from the HEL.

Recently the scalar fields (quintessence, phantom and tachyon) with potentials responsible for the evolution of the universe under the HEL within the framework of FRW space-time in general relativity has been determined by [42]. Anisotropic Bianchi type-V model with HEL has been studied by [43-45] have extended their study in different theories of gravitation. Recently [46] studied the HEL in the framework of Kontowski-Sachs cosmological models.

In this paper, the exact solutions of the Einstein's field equations for the Kaluza-Klein metric have been obtained by assuming Hybrid Expansion Law (HEL). The physical and geometrical properties of the model are also discussed. The paper is organized as follows: the metric and the field equations are presented in section 2. Section 3 deals with the isotropization and the solutions for the Hybrid Expansion Law. Finally the discussions and the conclusions are given in section 4 and 5 respectively.

2. Metric and Field equations:

The Kaluza-Klein type metric is given by

$$ds^2 = dt^2 - A^2\left(dx^2 + dy^2 + dz^2\right) - B^2 d\Psi^2, \tag{1}$$

where A and B are functions of cosmic time t only.

Here we are dealing only with an anisotropic fluid whose energy-momentum tensor is in the following form

$$T_v^u = diag\left[T_0^0, T_1^1, T_2^2, T_3^3, T_4^4\right]$$

We parameterize it as follows:

$$T_v^u = diag\left[\rho, -p_x, -p_y, -p_z, -p_\psi\right] = diag\left[1, -\omega_x, -\omega_y, -\omega_z, \omega_\psi\right]\rho$$

$$= diag\left[1, -\omega, -\omega, -\omega - (\omega + \delta)\right]\rho \tag{2}$$

where ρ is the energy density of the fluid; p_x, p_y, p_z and p_ψ are the pressures and $\omega_x, \omega_y, \omega_z$ and ω_ψ are the directional equation of state (EoS) parameters of the fluid. Now, parametrizing the deviation from isotropy by setting $\omega_x = \omega_y = \omega_z = \omega$ and then introducing skewness parameter δ that is the deviation from ω on ψ axis. Here ω and δ are not necessarily constants and can be functions of the cosmic time t.

The Einstein field equations, (in natural limits $8\pi G = 1$ and $c = 1)$ are

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -T_{\mu\nu} \tag{3}$$

where $g_{\mu\nu} u^\mu u^\nu = 1; u^\mu = (1,0,0,0,0)$ is the velocity vector; $R_{\mu\nu}$ is the Ricci tensor; R is the Ricci scalar, $T_{\mu\nu}$ is the energy-momentum tensor.

In a co-moving coordinate system, Einstein's field equations (3), for the anisotropic Kaluza-Klein space-time (1), with equation (2) yield

$$3\frac{\dot{A}\dot{B}}{AB} + 3\frac{\dot{A}^2}{A^2} = \rho \tag{4}$$

$$2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} = -\omega\rho \tag{5}$$

$$3\frac{\ddot{A}}{A} + 3\frac{\dot{A}^2}{A^2} = -(\omega + \delta)\rho \quad , \tag{6}$$

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where the overhead dot $(\cdot)$ denote derivative with respect to the cosmic time t .

3. Isotropization and the solution:

The spatial volume is given by

$$V = a^4 = A^3 B \tag{7}$$

where a is the mean scale factor.

Subtracting equation (5) from equation (6), we get

$$\frac{d}{dt}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\frac{\dot{V}}{V} = -\delta\rho \quad .$$

This on integrating gives

$$\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = \frac{\lambda}{V} \quad e^{\int \frac{\delta\rho}{\left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A}\right)} dt} \quad , \tag{8}$$

where λ is constant of integration.

In order to solve the above equation (8) we use the condition

$$\delta = \left(\frac{\frac{\dot{B}}{B} - \frac{\dot{A}}{A}}{\rho}\right) \quad . \tag{9}$$

Using equation (9) in the equation (8), we obtain

$$\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = \frac{\lambda}{V} \quad e^t \; . \tag{10}$$

In order to solve the system completely we impose the Hybrid expansion law (HEL) as

$$a = a_1 t^{\alpha} e^{\beta t} , \tag{11}$$

where $a_1 > 0$, $\alpha > 0$ and $\beta > 0$ are constants.

$$V = a^4 = A^3 B$$

$$V = {a_1}^4 t^{4\alpha} e^{4\beta t} \; .$$

The volumetric deceleration parameter is

$$q = -\frac{a\ddot{a}}{\dot{a}^2}$$

$$q = -1 + \frac{\alpha}{(\alpha + \beta t)^{2}} \quad . \tag{12}$$

Using equation (11) equation (10) implies that, we get

$$\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)=\frac{\lambda}{V}\quad e^{t}\quad .$$

This on integration gives

$$A = Bb\,c_1\,e^{\lambda\int e^{t}V^{-1}dt} \quad . \tag{13}$$

Using equation (11), we get the values of scale factors as

$$A = c_1^{\frac{1}{4}} a_1\, t^{\alpha} e^{\beta t}\,\exp\left[\frac{\lambda\, a_1^{-4}}{4}\int t^{-4\alpha} e^{-(4\beta-1)t} dt\right] \tag{14}$$

$$B = c_1^{-\frac{3}{4}} a_1\, t^{\alpha} e^{\beta t}\,\exp\left[-\frac{3\lambda\, a_1^{-4}}{4}\int t^{-4\alpha} e^{-(4\beta-1)t} dt\right] \tag{15}$$

Equation (14) and (15) can be written as

$$A = c_1^{\frac{1}{4}} a_1\, t^{\alpha} e^{\beta t}\,\exp\left[-\frac{\lambda\, a_1^{-4}}{4}(4\beta-1)^{4\alpha-1}\gamma\left[1-4\alpha,(4\beta-1)t\right]\right] \tag{16}$$

$$B = c_1^{-\frac{3}{4}} a_1\, t^{\alpha} e^{\beta t}\,\exp\left[\frac{3\lambda\, a_1^{-4}}{4}(4\beta-1)^{4\alpha-1}\gamma\left[1-4\alpha,(4\beta-1)t\right]\right] \tag{17}$$

where γ denotes the lower incomplete gamma function. The directional Hubble parameters are in the direction of x, y, z and ψ respectively.

$$H_x = H_y = H_z = \frac{\dot{A}}{A} = \left(\frac{\alpha}{t}+\beta\right) + \frac{\lambda\, a_1^{-4}}{4} t^{-4\alpha} e^{-(4\beta-1)t} \tag{18}$$

$$H_{\psi} = \frac{\dot{B}}{B} = \left(\frac{\alpha}{t}+\beta\right) - \frac{3\lambda\, a_1^{-4}}{4} t^{-4\alpha} e^{-(4\beta-1)t} \tag{19}$$

The mean Hubble parameter is given by

$$H = \frac{1}{4}\left(\frac{3\dot{A}}{A}+\frac{\dot{B}}{B}\right)$$

$$H = \frac{\alpha}{t}+\beta \tag{20}$$

The anisotropic parameter of the expansion (Δ) is defined as

$$\Delta = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2$$

where $H_i(i=1,2,3,4)$ represent the directional Hubble parameters in the directions of x , y, z and ψ respectively and is found as

$$\Delta = \frac{3\lambda^2 a_1^{-8} t^{-8\alpha} e^{-2(4\beta-1)t}}{16\left(\frac{\alpha}{t}+\beta\right)^2} \tag{21}$$

The expansion scalar θ is given by

$$\theta = 4H = 4\left(\frac{\alpha}{t}+\beta\right). \tag{22}$$

The shear scalar σ^2 is given by

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{4} H_i^2 - 4H^2\right) = \frac{4}{2}\Delta H^2$$

$$\sigma^2 = \frac{3}{8}\lambda^2 a_1^{-8} t^{-8\alpha} e^{-2(4\beta-1)t} \tag{23}$$

Using equations (18) and (19) in equation (4), we obtain the energy density of the fluid as

$$\rho = 6\left(\frac{\alpha}{t}+\beta\right)^2 - \frac{3}{8}\lambda^2 a_1^{-8} t^{-8\alpha} e^{-2(4\beta-1)t} \tag{24}$$

Using equations (18), (19) and (23) in equation (9), we obtain the deviation parameter as

$$\delta = \frac{\lambda a_1^{-4} t^{-4\alpha} e^{-2(4\beta-1)t}}{\frac{3}{8}\lambda^2 a_1^{-8} t^{-8\alpha} e^{-2(4\beta-1)t} - 6\left(\frac{\alpha}{t}+\beta\right)^2} \tag{25}$$

Using equations (18), (19), (23) and (24) in equation (6), we obtain the deviation-free parameter as

$$\omega = -\left\{\frac{6\left(\frac{\alpha}{t}+\beta\right)^2 - \frac{3\alpha}{t^2} + \frac{3}{8}\lambda^2 a_1^{-8} t^{-8\alpha} e^{-2(4\beta-1)t} + \lambda a_1^{-4} t^{-4\alpha} e^{-(4\beta-1)t}}{6\left(\frac{\alpha}{t}+\beta\right)^2 - \frac{3}{8}\lambda^2 a_1^{-8} t^{-8\alpha} e^{-2(4\beta-1)t}}\right\} \tag{26}$$

4. Discussion:

- From *fig. (i).* it is observed that in this model anisotropy parameter (Δ) decreases to zero very quickly. Hence, the model reaches to isotropy after some finite time which matches with the recent observations as the universe is isotropic at large scale.

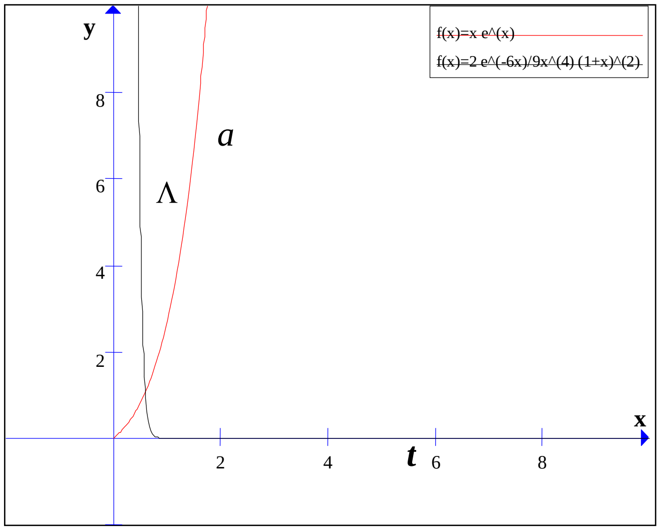

Fig. (i): Evolution of average scale factor and anisotropy parameter (Δ) *for* $l = a_1 = \alpha = \beta = 1$.

- In this model with HEL, we obtained the time dependent deceleration parameter q. Also, the early deceleration and late time acceleration takes place at $t = \dfrac{\sqrt{\alpha} - \alpha}{\beta}$.

From *fig. (ii)*, one can observe that $q \to -1$ as $t \to \infty$. For $\alpha = \dfrac{2}{3}$ the deceleration parameter q is in the range $-1 \le q \le 0.5$ which is consistent with the observations that the present day universe is undergoing accelerated expansion [1-2].

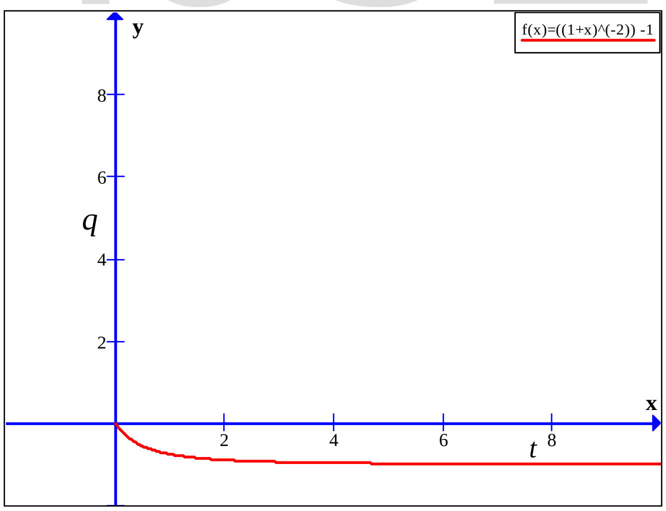

Fig. (ii): Variation of deceleration parameter (ρ) *Vs* time (t) for $\alpha = \beta = 1$

- The equation of state parameter ω attains the value $\omega = -1$ after some finite t i.e. the model approaches to ΛCDM model after some finite t. The available data sets in cosmology, especially the SNe Ia data [3-4] the SDSS data [47] the three year WMAP data [9] and [48] all indicate that the ΛCDM model or the model that reduces to ΛCDM are serves as a standard model in cosmology, is an excellent model to describe the cosmological evolution.

5. Conclusion:-

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Here we have studied the Kaluza-Klein cosmological models in which the solutions of the field equations are obtained under the assumption of the Hybrid Expansion Law. We observed that the hybrid expansion law gives time dependent deceleration parameter representing a transitioning Universe from early decelerating phase to current accelerating phase. Also, in the resulting model the anisotropy of expansion dies out very quickly and attains isotropy after some finite time.

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